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# STRENGTH OF SUBDIVISION GRAPH CERTAIN FUZZY GRAPHS

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**ABSTRACT**

In this paper we find the strength of subdivision graph of strong fuzzy path, strong fuzzy butterfly graph, strong fuzzy star graph, strong fuzzy Bull graph, strong fuzzy cycle, strong fuzzy diamond graph, fuzzy complete graph.



## INTRODUCTION

We introduce, in this paper, the concept of extra strong path to find the strength of subdivision graph of various strong fuzzy graphs. The notion of a fuzzy subset was introduced for the first time in 1965 by Lofti A. Zadeh [10]. Azriel Rosenfeld [6] in 1975, defined the fuzzy graph based on definitions of fuzzy sets and relations. He was the one who developed the theory of fuzzy graphs. The concept of strength of connectivity between two vertices of a fuzzy graph was introduced by M. S. Sunitha [7] and extended by Sheeba M. B. [8], [9] to arbitrary fuzzy graphs. Sheeba called it, strength of the fuzzy graph and determined it, in two different ways, of which one is by introducing weight matrix of a fuzzy graph and other by introducing the concept of extra strong path between its vertices.

### *Preliminaries*

A fuzzy graph $G=(V,\mu,\sigma)$ [6] is a nonempty set $V$ together with a pair of functions $\mu:V\to[0,1]$ and $\sigma:V\times V\to[0,1]$ such that for all $u,v\in V$, $\sigma(u,v)=\sigma(uv)\le\mu(u)\wedge\mu(v)$. We call $\mu$ the fuzzy vertex set of $G$ and $\sigma$ the fuzzy edge set of $G$. Here after we denote the fuzzy graph $G(\mu,\sigma)$ simply by $G$ and the underlying crisp graph of $G$ by $G(V,E)$ with $V$ as vertex set and $E=\{(u,v)\in V\times V:\sigma(u,v)>0\}$ as the edge set or simply by $G^*$. An edge $u\,v$ is strong [1] if and only if $\sigma(uv)=\mu(u)\wedge\mu(v)$. A fuzzy graph $G$ is complete [5] if $\sigma(uv)=\mu(u)\wedge\mu(v)$ for all $u,v\in V$. A fuzzy graph $G$ is a strong fuzzy graph [4] if $\sigma(uv)=\mu(u)\wedge\mu(v)$, $\forall uv\in E$.

A path $P$ of length $n-1$ in a fuzzy graph $G$ \cite{ar} is a sequence of distinct vertices $v_1,v_2,v_3,\ldots,v_n$, such that $\sigma(v_i,v_{i+1})>0$, $i=1,2,3,\ldots,n-1$. If $v_1=v_n$ and $n\ge3$ we call $P$ a cycle and cycle $P$ is called a fuzzy cycle if it contains more than one weakest edge. The strength of a path in a fuzzy graph is defined as the weight of the weakest edge of the path [6] which is $\wedge_{i=1}^{n}\sigma(v_{i-1}v_i)$. A path $P$ is said to connect the vertices u and v of G strongly if its strength is maximum among all paths between $u$ and $v$. Such paths are called strong paths [5]. Any strong path between two distinct vertices $u$ and $v$ in $G$ with minimum length is called an extra strong path between them [8]. There may exists more than one extra strong paths between two vertices in a fuzzy graph $G$. But, by the definition of an extra strong path each such path between two vertices has the same

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length. The maximum length of extra strong paths between every pair of distinct vertices in $G$ is called the strength of the graph $G$ [9].

For a fuzzy graph $G$, if $G^*$ is the path $P = v_1 v_2 \ldots v_n$ on $n$ vertices then the strength of the graph $G$ is its length $(n-1)$ [8].

Here after we denote the strength of a fuzzy graph $G$ by $\mathrm{S}(G)$.

Theorem 1: [9] If $G$ is a complete fuzzy graph then $\mathrm{S}(G)$ is one.

The following theorems determine the strength of a fuzzy cycle in terms of the order of its crisp graph and the number of weakest edges.

Theorem 2: [8] In a fuzzy cycle $G$ of length $n$, suppose there are $l$ weakest edges where $l \le [\frac{n+1}{2}]$. If these weakest edges altogether form a subpath then $\mathrm{S}(G)$ is $n-l$.

Theorem 3: [8] Let $G$ be a fuzzy cycle with crisp graph $G^*$ a cycle of length $n$, having $l$ weakest edges which altogether form a subpath. If $l > [\frac{n+1}{2}]$, then $\mathrm{S}(G)$ is $[\frac{n}{2}]$.

Theorem 4: [8] Let $G$ be a fuzzy cycle with crisp graph $G^*$ a cycle of length $n$, having $l$ weakest edges which do not altogether form a subpath. If $l > [\frac{n}{2}]-1$ then the strength of the graph is $[\frac{n}{2}]$ Definition 5: [2] A finite sequence of fuzzy graphs $G_1, G_2, \ldots, G_m$ with the property that $V(G_i) \cap V(G_j)$ is nonempty only if $|j-i| \le 1, 1 \le i, j \le m$ is called a properly linked sequence or simply properly linked. It is $n-$ linked if the crisp graph induced by $V(G_i) \cap V(G_j)$ is $K_n$, a complete graph on $n$ vertices, if $|j-i| = 1, 1 \le i, j \le m$.

***Main Results***

Definition 6: Let $G(V, \mu, \sigma)$ be a fuzzy graph with underlying crisp graph $G(V, E)$. Then the subdivision graph of $G$, denoted by $sd(G)$ is the fuzzy graph $sd(G)(V_{sd}, \mu_{sd}, \sigma_{sd})$ with the underlying crisp graph is the subdivision graph of $G(V, E)$, where the vertex set $V_{sd} = V \cup E$ and the membership functions $\mu_{sd}$ and $\sigma_{sd}$ are defined as

$$\mu_{sd}(u) = \begin{cases} \mu(u) & \text{if } u \in V \\ \sigma(u) & \text{if } u \in E \end{cases} \quad \text{and}$$

$$\sigma_{sd}(u,e) = \begin{cases} \mu_{sd}(u) \wedge \mu_{sd}(e) & \text{if } u \in V, e \in E \text{ and } u \text{ lies on } e \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 7: Let $G$ be a strong fuzzy graph with its underlying crisp graph is a path on $n$ vertices. Then the strength $\mathrm{S}(sd(G))$ of the subdivision graph $sd(G)$ of $G$ is $2\mathrm{S}(G)$.

Proof:

The subdivision graph of a strong fuzzy graph with its underlying graph is a path on $n$ vertices is a strong fuzzy graph with its underlying crisp graph is a path on $2n-1$ vertices (See Figure 1) So strength of $sd(G)$ is $(2n-1)-1 = 2(n-1) = 2\mathrm{S}(G)$.

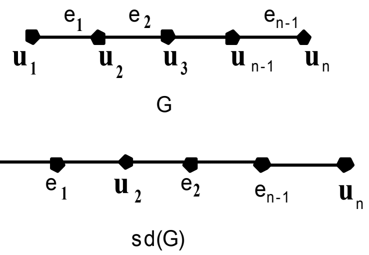


**Figure 1** A strong fuzzy path graph and its subdivision graph

Theorem 8: Let $G$ be a strong fuzzy butterfly graph. Then the strength $\mathrm{S}(sd(G))$ of the subdivision graph of $G$ is $6$.

Proof:

Let the vertex set of $G$ be $\{u_1, u_2, u_3, u_4, u_5\}$ with $u_3$ as the central vertex. Then $G$ is a $1-$ linked fuzzy graph with two parts $G_1$ and $G_2$, where both $G_1$ and $G_2$ are fuzzy cycles on $3$ vertices.

Its subdivision graph is also a $1-$ linked fuzzy graph with two parts which are cycles on $6$ vertices (See Figure 2 Since each part $G_i, i = 1,2$ of $G$ has atleast two weakest edges of $G_i$, $sd(G_i), i = 1,2$ has at least $4$ weakest edges in $sd(G_i)$.

Let u, v be any two vertices of $sd(G)$. If both $u$ and $v \in V(sd(G_i)), i = 1,2$ then any extra strong path joining $u$ and $v$ lie completely in $sd(G_i), i = 1$ or $2$. So the strength of the $u-v$ path in $G$ is $3$ by Theorem 4. Since $u_3 \in V(G_1) \cap V(G_2)$, $u_3 \in V(sd(G_1)) \cap V(sd(G_2))$.If $u \in sd(G_1)$, $\{u_3\}$ and $v \in sd(G_2)$, $\{u_3\}$ then all the $u-v$ paths can be considered as the sum of two paths $P_1$ of $sd(G_1)$ joining

$u$ to $u_3$ and $P_2$ of $sd(G_2)$ joining $u_3$ to $v$. Therefore, the length of any extra strong the $u-v$ path is less than or equal to the length of any extra strong $u-u_3$ path in $sd(G_1)$ and $u_3-v$ path in $sd(G_2)$ which is less than or equal to $3+3=6$.

Also when $u$ is the vertex $\in V(sd(G))$ corresponding to the edge $u_1u_2$ in $G_1$ and $v$ is the vertex $\in V(sd(G))$ corresponding to the edge $u_4u_5$ in $V(G_2)$ the strength of the $u-v$ path is exactly $6$.
Hence the theorem.

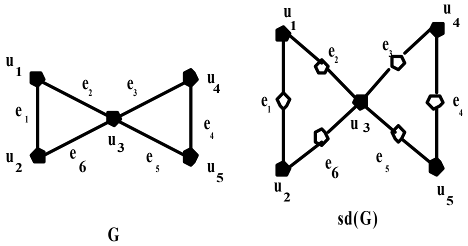


**Figure 2** A strong fuzzy butterfly graph and its subdivision graph

Theorem 9: Let $G$ be a strong fuzzy Bull graph then the strength $S(sd(G))$ of the subdivision graph of $G$ is $6$.
Proof

A fuzzy bull graph $G$ is a $1-$linked fuzzy graph with three parts. Let $P, P'$ and $P''$ be its parts, where $P$ and $P''$ are fuzzy paths on two vertices and $P'$ is a fuzzy triangle. Then $sd(G)$ is also a $1-$ linked fuzzy graph with parts $G_1 = sd(P), G_2 = sd(P')$ and $G_3 = sd(P'')$ (See Figure 3).

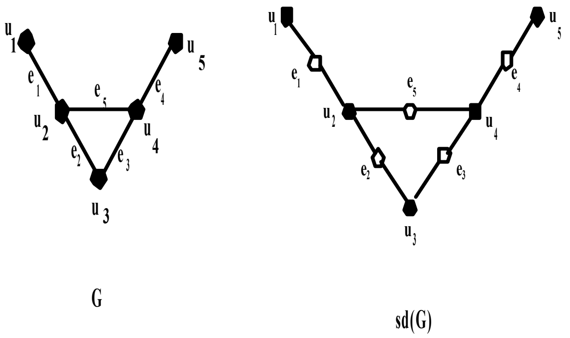


**Figure 3** A strong fuzzy Bull graph and its subdivision graph

Let $u$ and v be any two non-adjacent vertices of $sd(G)$. If $u, v \in V(G_1)$ or $u, v \in V(G_3)$ then the length of any extra strong $u-v$ path in $G$ is $2$. Since both $sd(P)$ and $sd(P'')$ are paths on $3$ vertices.

If $u, v \in V(G_3)$ then all the paths joining $u$ and $v$ lie completely in $G_3$. Since $G_3$ is the subdivision graph of the strong fuzzy triangle $P'$, it is a strong fuzzy cycle on $6$ vertices. As $P'$ contains at least $2$ weakest edges, $sd(G_3)$ contains atleast $4$ weakest edges. Therefore by Theorem 3 the length of the extra strong of $u-v$ path in $G_3$ is 3.

Let $\{w\} = V(G_1) \cap V(G_3)$ and $\{w'\} = V(G_2) \cap V(G_3)$. If $u \in V(G_1)$ and $v \in V(G_2)$ then all the $u-v$ paths pass through both $w$ and $w'$ in $sd(G)$. Since $w$ and $w'$ are adjacent in $G$, the extra strong path joining $w$ and $w'$ in $sd(G)$ is $wew'$ where $e$ is the vertex in $sd(G)$ corresponding to the edge $ww'$ in $G$. So the length of the extra strong path joining $u$ and $v$ is $\leq 2+2+2=6$. When $u$ and $v$ are the pendant vertices of $G$ then the extra strong $u-v$ path has length exactly $6$. Therefore $S(G)=6$.

For a fuzzy tree $G$, the strength of \$G\$ is the diameter of the underlying crisp graph of $G$. The subdivision graph of a fuzzy star graph is a fuzzy tree. From this we have the following Theorem.

Theorem 10: The strength of the subdivision graph of a fuzzy star graph $G$ is $4$.

Note 11:

Let $G$ be a strong fuzzy cycle on $n$ vertices with $l$ weakest edges in $G$ having weight $w$. Then the edges in $sd(G)$ incident with the vertex corresponding to each weakest vertex and the edges incident with that weakest vertex are of weight $w$. Therefore in $sd(G)$, there are 2l weakest edges.

Theorem 12: Let $G$ be a strong fuzzy cycle of length $n$, which contains $l$ weakest edges and also let $s$ denote the maximum length of the subpath which does not contain any weakest edge of $G$ and is of strength $S(G)$.
Then the strength $S(sd(G))$ of the subdivision graph of $G$ is $2S(G)$.
Proof:

We have by Note 11 for a strong fuzzy cycle $G$ of length $n$, if there are $l$ weakest edges which altogether form a subpath in $G$ then there are 2l weakest edges which altogether form a subpath in $sd(G)$. We have by Theorem 2 if $2l \leq [\frac{2n+1}{2}]$ then $S(sd(G)) = 2n-2l = 2(n-l)$.

If $2l \leq [\frac{2n+1}{2}]$ then $l \leq [\frac{n+1}{2}]$ so $S(sd(G)) = 2S(G)$. Also by Theorem 4 if $2l > [\frac{2n+1}{2}]$ then $S(sd(G)) = [\frac{2n}{2}] = 2[\frac{n}{2}]$. We have $2l > [\frac{2n+1}{2}]$ implies $l \geq [\frac{n+1}{2}]$ so $\mathrm{S}(sd(G)) = 2\mathrm{S}(G)$.

Suppose there are $l$ weakest edges which do not altogether form a subpath in $G$, then the $21$ weakest edges of $sd(G)$ also do not form a subpath in $sd(G)$.

So by Theorem 4 if $2l > [\frac{2n}{2}] - 1$ then

$\mathrm{S}(sd(G)) = [\frac{2n}{2}]$.

We have $2l > [\frac{2n}{2}] - 1$ implies $l > [\frac{n}{2}] - 1$. Hence, if $l > [\frac{n}{2}] - 1$ then $\mathrm{S}(sd(G)) = 2\mathrm{S}(G)$.

Similarly if $2l < [\frac{2n}{2}] - 1$ then $\mathrm{S}(sd(G)) = [\frac{2n}{2}]$. Also $2l < [\frac{2n}{2}] - 1$ implies $l \leq [\frac{n}{2}] - 1$. So $\mathrm{S}(sd(G)) = [\frac{2n}{2}] = 2\mathrm{S}(G).$

Hence the proof.

Theorem 13: The strength of the subdivision graph of a strong fuzzy diamond graph is $4$.

Proof

Let $G$ be a strong fuzzy diamond graph. Also let $u$ and $v$ be two non-adjacent vertices of $sd(G)$.

Case 1: $u, v \in V(G)$.

If $u$ and $v$ are adjacent in $G$ and $e$ be the edge joining $u$ and $v$ in $G$ then the strength of the $u-v$ path is less than or equal to $\mu_{sd}(u) \wedge \mu_{sd}(v)$ and which is equal to $\sigma_{sd}(e)$. So the path $u\ e\ v$ is the extra strong path joining $u$ and $v$ in $sd(G)$ which is of length $2$.

If $u$ and $v$ are non-adjacent vertices in $G$ then $u, v \in \{u_2, u_4\}$ as shown in Figure 4.

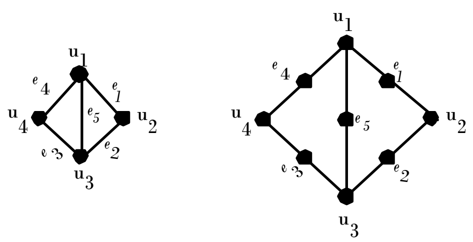


**Figure 4** A strong fuzzy diamond graph $G$ and its subdivision graph

Then any extra strong path joining $u$ and $v$ in $sd(G)$ pass either through $u_1$ or through $u_3$ depending up on their weights. Without loss of generality assume that $\mu(u_1) \geq \mu(u_3)$. Since $u_1$ is adjacent to both $u$ and $v$ and $e_1$ is the edge $uu_1$ and $e_4$ is the edge $u_1v$ in $G$, so $ue_1u_1e_4v$ is an extra strong path and it is of minimum length among all the other strong paths and is of length $4$.

Case 2: $u, v \in E(G)$.

If $u$ and $v$ have a common vertex $w$ in $G$ then in $sd(G)$ the path $u\ w\ v$ is an extra strong path since, the strength of all the paths joining $u$ and $v$ in $sd(G)$ have strength $\leq \mu_{sd}(u) \wedge \mu_{sd}(v)$ and $\mu_{sd}(w) \geq \mu_{sd}(u) \wedge \mu_{sd}(v)$. So the length of the extra strong path joining $u$ and $v$ is $2$.

Otherwise, suppose $u$ and $v$ have no common vertex in $G$ then $u$ and $v \in \{e_1, e_3\}$ or $\{e_2, e_4\}$ (See Figure \ref{sd(D)}). Without loss of generality assume that $u$ and $v \in \{e_1, e_3\}$. In this case all the $u-v$ paths have strength less than or equal to $\mu_{sd}(u) \wedge \mu_{sd}(v) = \mu_{sd}(u_1) \wedge \mu_{sd}(u_2) \wedge \mu_{sd}(u_3) \wedge \mu_{sd}(u_4)$. Therefore, the length of the extra strong path is the minimum distance between $u$ and $v$ which is $4$.

Case 3: $u \in V(G)$ and $v \in E(G)$ in $sd(G)$.

Without loss of generality assume that $u = u_1$ and $v = e_3$ where $e_3$ is the vertex in $sd(G)$ corresponding to the edge $u_3u_4$ in $G$ (See Figure 4) Then strength of the $u-v$ path in $sd(G)$ $\leq \mu_{sd}(u) \wedge \mu_{sd}(v) = \mu(u_1) \wedge \mu(u_3) \wedge \mu(u_4)$. So the extra strong path joining $u$ and $v$ lies completely in the maximal partial fuzzy subgraph of $sd(G)$ with vertex set $\{u_1, u_3, u_4, e_3, e_4, e_5\}$, which is a strong fuzzy cycle on $6$ vertices. So the length of the extra strong path joining $u$ and $v$ is $3$.

Theorem 14: Let $G$ be a fuzzy complete graph. Then the strength $\mathrm{S}(sd)(G)$ is $3$ for $n = 3$ and $4$ for $n > 3$.

Proof

When $n = 3$, $sd(G)$ is a strong fuzzy cycle on $6$ vertices having at least $4$ weakest edges. So the the result follows by the Theorem 4.

Consider the case, $n > 3$. Let $u$, $v$ be two non-adjacent vertices of $sd(G)$.

If $u$ and $v$ are the vertices of $G$ then the extra strong path joining $u$ and $v $ is $uev$ where $e$ is the edge $u$ $v$ in $G$ and is of length $2$.

If $u$ and $v$ are edges of $G$ then, if they have a common vertex $w$ in $G$ then the path $u\ w\ v$ in $sd(G)$ is of strength exactly equal to $\mu_{sd}(u) \wedge \mu_{sd}(v)$, which is an extra strong path joining them in $sd(G)$.

Suppose $u$ and $v$ are edges of $G$ and have no vertex in common. Let $u = u_j u_k$ and $v = u_l u_m$ be that vertices in $sd(G)$. The weight of the edges $uu_j$ and $uu_k$ are the same and the weight of the edges $vu_l$ and $vu_m$ are the same in $sd(G)$ and the extra strong path joining any two vertices $u'$ and $u''$ of $G$ in $sd(G)$ is $u'eu''$, where $e$ is the edge joining $u'$ and $u''$ in $G$. So all the $u-v$ paths have same strength in $sd(G)$. Therefore, the length of the extra strong path joining $u$ and $v$ is the shortest length joining $u$ and $v$ in $sd(G)$ which is $4$.

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